MAC2312 **Integrals Review** **Dr. Irma Cruz-White**

1) $\int xe^{3x}dx$ $= \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$ Integration by Parts

2) $\int x^2 \ln x dx$ $= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$ Integration by Parts

3) $\int \frac{x^2 - 3x + 1}{x^3}dx$ $= \ln|x| + \frac{3}{x} - \frac{1}{2x^2} + C$ Term by Term

4) $\int \frac{e^{\frac{1}{x}}}{x^2}dx$ $= -e^{\frac{1}{x}} + C$ Substitution

5) $\int \frac{1}{x^2 + 6x + 20}dx$ $= \frac{1}{\sqrt{11}}\arctan\frac{x+3}{\sqrt{11}} + C$ Complete square, subs. $u = x+3$, $du = dx$, basic integral

6) $\int \frac{1}{3x\sqrt{9x^2 - 25}}dx$ $= \frac{1}{3}\cdot\frac{1}{5}\operatorname{arcsec}\frac{|3x|}{5} + C$ substitution, arcsec.

$u = 3x$
$du = 3\,dx \Rightarrow \frac{1}{3}du = dx$

7) $\int x^3\sqrt{16 - 4x^2}dx$ $= 64\int \sin^3\theta\cos^2\theta\,d\theta$ $= -\frac{64}{3}\left(\frac{\sqrt{16-4x^2}}{4}\right)^3 + \frac{64}{5}\left(\frac{\sqrt{16-4x^2}}{4}\right)^5 + C$

$a = 4,\ u = 2x$
$\sin\theta = \frac{2x}{4} \Rightarrow x = 2\sin\theta$

Trig. substitution, Trig. Integral

8) $\int \frac{1}{9 - x^2}dx$ $= \frac{-\ln|3-x|}{6} + \frac{\ln|3+x|}{6} + C$ Partial Fractions

9) $\int \tan^3 x \sec^3 x\,dx$ $= \frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$ Trig. Integral

$\tan^2 x \sec^2 x \tan x \sec x$

10) $\int_4 \frac{1}{\sqrt{4 - x}}dx$ $= \lim_{b\to 4}\int_0^b \frac{1}{\sqrt{4-x}}dx$ $= 4$ improper integral

11) $\int_{-\infty}^{1} \frac{1}{\sqrt[3]{x}}\,dx$ improper integral

$\lim_{a\to-\infty} \int_{a}^{-1} (x^{1/3})\,dx$

12) $\int \frac{x^2-3x+1}{\sqrt{x}}\,dx$ term by term

13) $\int x^2 e^{x^3}\,dx$ substitution

14) $\int \frac{1}{x^2+10x+30}\,dx$ complete square

15) $\int \frac{1}{\sqrt{25-9x^2}}\,dx$ substitution, arcsin

$u=3x \Rightarrow du=3dx$

$a=5 \qquad \frac{1}{3}du=dx$

16) $\int \frac{x^3}{\sqrt{4x^2+1}}\,dx$ Trigonometric substitution, trig. integral

$u=2x$

$a=1$ $\qquad \tan\theta=2x$

17) $\int \frac{1}{4-x^2}\,dx$ partial fractions

18) $\int \sin^3 x \cos^2 x\,dx$ Trig. Integral

19) $\int_{4}^{6} \frac{1}{\sqrt{x-4}}\,dx$ improper integral

$\lim_{a\to 4} \int_{a}^{6} \frac{1}{\sqrt{x-4}}\,dx = \ldots$

20) $\int_{1}^{\infty} \frac{1}{\sqrt[3]{x}}\,dx$ improper integral

$\lim_{b\to\infty} \int_{1}^{b} x^{-1/3}\,dx, \ldots$